AP Calculus AB
Related Rates Worksheet #3

Name Key

1. Sand is pouring from a pipe at a rate of 22.5 cubic feet per second. The falling sand forms a conical pile on the ground. The altitude of the pile is always 2/3 the diameter of the base.
   a) What is the rate of change of the altitude at the instant the altitude is 6 feet?
   b) What is the rate of change of the height of the pile at the instant that area of the base of the pile is 24π square feet?
   c) What is the height of the pile at the instant the rate of change of the volume is equal to the rate of change of the area of the base?
   d) What is the height of the pile when the rate of change of the volume is equal to the rate of change of the height of the pile?

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

$\frac{dV}{dt} = 22.5 \text{ ft}^3/\text{sec}$

$h = \frac{2}{3}d$

$h = \frac{2}{3}(2r)$

always true → $\left[\begin{array}{l} h = \frac{4r}{3} \\ r = \frac{3h}{4} \end{array}\right]$

a) $\frac{dh}{dt} = ?$

$h = 6$ ft.

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{3h}{4}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{9h^2}{16} \cdot h$

$V = \frac{3\pi}{16} h^3$

$\frac{dV}{dt} = \frac{9\pi}{16} h^2 \frac{dh}{dt}$

$(22.5) = \frac{9\pi}{16}(6)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = .354$ ft/sec.

b) $\frac{dh}{dt} = ?$

$A = 24\pi \text{ ft}^2$

$24\cancel{\pi} = \cancel{\pi} r^2$

$r = \sqrt{24}$

given: $\left[h = \frac{4r}{3}\right]$

$h = \frac{4\sqrt{24}}{3}$

$\frac{dV}{dt} = \frac{9\pi}{16} h^2 \frac{dh}{dt}$

$(22.5) = \frac{9\pi}{16}\left(\frac{4\sqrt{24}}{3}\right)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = .298$ ft/sec

c) $h = ?$

$\frac{dV}{dt} = \frac{dA}{dt}$

$\frac{\cancel{9\pi}}{16} h^{\cancel{2}} \cancel{\frac{dh}{dt}} = \frac{\cancel{9\pi}}{8} \cancel{h} \cancel{\frac{dh}{dt}}$

$\frac{h}{16} = \frac{1}{8}$

$8h = 16$

$h = 2$ ft.

$A = \pi r^2$

$A = \pi\left(\frac{3h}{4}\right)^2$

given: $\left[r = \frac{3h}{4}\right]$

$A = \pi \cdot \frac{9h^2}{16}$

$A = \frac{9\pi}{16} h^2$

$\frac{dA}{dt} = \frac{9\pi}{8} h \frac{dh}{dt}$

d) $\frac{dV}{dt} = \frac{dh}{dt}$

$\frac{9\pi}{16} h^2 \cancel{\frac{dh}{dt}} = \cancel{\frac{dh}{dt}}$

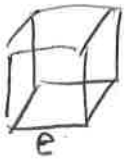
$\frac{9\pi}{16} h^2 = 1$

$h^2 = \frac{16}{9\pi}$

$h = \frac{4}{3\sqrt{\pi}}$

$h = .75$ ft.

2. The edge of a cube is expanding at a rate of 1.25 centimeters per second.
   a) How fast is the volume changing when the length of an edge is 12 centimeters?
   b) How fast is the surface area changing at the same instant?
   c) What is the length of the edges of the cube at the instant the rate of change of the volume is equal to the rate of change of the surface area?



$\frac{de}{dt} = 1.25$ cm/sec

a) $\frac{dV}{dt} = ?$

$e = 12$ cm

$V = e^3$

$\frac{dV}{dt} = 3e^2 \frac{de}{dt}$

$\frac{dV}{dt} = 3(12)^2(1.25)$

$\frac{dV}{dt} = 540$ cm³/sec

b) $\frac{dA}{dt} = ?$

$A = 6e^2$

$\frac{dA}{dt} = 12e \frac{de}{dt}$

$\frac{dA}{dt} = 12(12)(1.25)$

$\frac{dA}{dt} = 180$ cm²/sec

c) $e = ?$

$\frac{dV}{dt} = \frac{dA}{dt}$

$3e^{\cancel{2}} \cancel{\frac{de}{dt}} = 12\cancel{e}\cancel{\frac{de}{dt}}$

$3e = 12$

$e = 4$ cm

3. A stone is dropped into a still pond and it produces circular ripples.
   a) If the radius of one of the ripples increases at a rate of 2.5 feet per second, how fast is the area of a ripple changing when the radius is 20 feet?
   b) If the area is changing at a rate of 14 square feet per second, how fast is the radius changing when the area is 110 square feet?
   c) What is the radius of the ripple at the instant the rate of change of the radius is equal to the rate of change of the area?

a) $\frac{dr}{dt} = 2.5$ ft/sec

$\frac{dA}{dt} = ?$

$r = 20$ ft.

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(20)(2.5)$

$\frac{dA}{dt} = 100\pi$ ft²/sec

b) $\frac{dA}{dt} = 14$ ft²/sec

$\frac{dr}{dt} = ?$

$A = 100$ ft²

$100 = \pi r^2$

$r = \sqrt{\frac{100}{\pi}}$ ft.

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$(14) = 2\pi\left(\sqrt{\frac{100}{\pi}}\right)\frac{dr}{dt}$

$\frac{dr}{dt} = .378$ ft/sec

c) $r = ?$

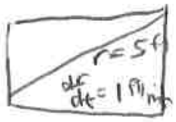
$\frac{dr}{dt} = \frac{dA}{dt}$

$\cancel{\frac{dr}{dt}} = 2\pi r \cancel{\frac{dr}{dt}}$

$1 = 2\pi r$

$r = \frac{1}{2\pi}$ ft.

4) The sides of a rectangle are increasing such that $\frac{dl}{dt} = 3\frac{dw}{dt}$ and the rate of change of the diagonals is $\frac{dr}{dt} = 1\frac{ft}{min}$. At the instant when the length is 4 feet and the width is 3 feet. Find the rate of change of the length.



$w = 3$ ft

$\frac{dw}{dt} =$

$l = 4$ ft.

$\frac{dl}{dt} = ?$

$$l^2 + w^2 = r^2$$

$$\cancel{2}l\frac{dl}{dt} + \cancel{2}w\frac{dw}{dt} = \cancel{2}r\frac{dr}{dt}$$

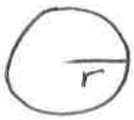
$$(4)\frac{dl}{dt} + (3)\left(\frac{1}{3}\frac{dl}{dt}\right) = (5)(1)$$

$$4\frac{dl}{dt} + \frac{dl}{dt} = 5$$

$$5\frac{dl}{dt} = 5$$

$$\frac{dl}{dt} = 1 \text{ ft/min}$$

5) The radius of a circle is increasing at a non-zero rate. At the instant the rate of change of the area of the circle is equal to the rate of change of the circumference of the circle, find the length of the radius.



$\frac{dA}{dt} = \frac{dC}{dt}$

$r = ?$

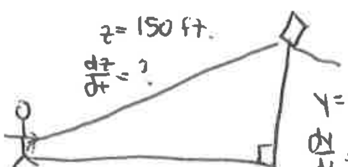
$$A = \pi r^2 \qquad C = 2\pi r$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} \qquad \frac{dC}{dt} = 2\pi\frac{dr}{dt}$$

$$\cancel{2\pi}r\cancel{\frac{dr}{dt}} = \cancel{2\pi}\cancel{\frac{dr}{dt}}$$

$r = 1$ unit

6) A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child letting out the cord when 150 feet of cord is out? (Assume that the cord forms a straight line.)



$z = 150$ ft.

$\frac{dz}{dt} = ?$

$y = 90$ ft

$\frac{dy}{dt} = 0$

$x = 120$ ft.

$\frac{dx}{dt} = 5$ ft/sec

$$x^2 + y^2 = z^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$$

$$(120)(5) + (90)(0) = (150)\frac{dz}{dt}$$

$$600 = 150\frac{dz}{dt}$$

$$\frac{dz}{dt} = 4 \text{ ft/sec.}$$

7) Water is being drained from a conical tank (vertex down) at a rate of 45 ft^3 / sec. The tank has a 40 foot diameter and is 50 feet deep. At what rate is the radius of the water's surface changing when the depth of the water is 15 feet?

$\frac{dV}{dt} = 45$ ft³/sec

$\frac{dr}{dt} = ?$

$h = 15$ ft.

$\frac{r}{h} = \frac{20}{50}$

$5r = 2h$

always true → $\left[\begin{array}{l} r = \frac{2h}{5} \\ h = \frac{5r}{2} \end{array}\right]$

$\frac{r}{15} = \frac{20}{50}$

$5r = 30$

$r = 6$ ft.

$$V = \frac{\pi}{3}r^2h$$

$$V = \frac{\pi}{3}r^2\left(\frac{5r}{2}\right)$$

$$V = \frac{5\pi}{6}r^3$$

$$\frac{dV}{dt} = \frac{5\pi}{2}r^2\frac{dr}{dt}$$

$$(45) = \frac{5\pi}{2}(6)^2\frac{dr}{dt}$$

$$45 = 90\pi\frac{dr}{dt}$$

$$\frac{1}{2\pi} \text{ ft/sec} = \frac{dr}{dt}$$